Assignment #1: Mission Analysis

**Due: Wednesday, Oct 12 at 5:00 P.M. ET**

***Prompt*** This assignment is framed as a written technical report. The introduction and prompts have been provided throughout this template. You are asked to complete the sections/tables/figures highlighted in red. Point values for each section are provided. You will be graded not only the correctness of solutions but also the clarity of presentation.

***Rules***

* Assignment must be completed by a group of 3. Turn in 1 report per group.
* All group members must contribute equally to the report. The same grade will be given to all the group members.
* Report must use this template and be typeset.
* Report must be submitted electronically through Canvas.
* Report must be written with complete sentences.
* All equations must be numbered and all variables in equations must be labeled.
* Submit any code (e.g. in Matlab) developed for this work as part of the Appendix. Clearly annotate the code.

***Rubric***

The following criteria will be used when evaluating the report:

* Reported values are within 15% of the solution key values
* Reported values have correct units
* Figure axes are labeled, and key parts of figures are annotated
* Report is written in a concise and understandable style
* Governing equations are appropriately used, explained, and numbered
* Discussion and conclusions reflect an understanding of the course material

Example of how to include equations in report

To calculate the payload mass, we use the rocket equation:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where denotes the wet mass of the spacecraft before the maneuver, is the final mass of the spacecraft after the maneuver, is the effective increment in velocity, is the gravitational constant of 9.8 m/s2, and is the specific impulse.

Since the maneuvers we consider are all performed with the same propulsion system in space, we can simply add all of the delta-v contributions, e.g.

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

We substitute Eq. 2 into Eq. 1 using the value of find

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

This mass represents the initial mass of the spacecraft before the burn is applied.

Trade Study for Crewed Mission to the Moon

Carter Briggs, Cole Helsel, Hunter Sagerer

1. **Nomenclature (5 pts)**

*Compile and define all the variables used in the report here, e.g.*

= Change in velocity increment (m/s)

= Specific impulse (s)

*T* = Thrust (N)

*M0(i)* = Initial mass of ith stage (kg)

.

1. **Introduction**

In the early 1960s, U.S. President John Kennedy set the ambitious goal of landing people on the moon by the end of the decade. This was an incredible undertaking given that at the time the U.S. had only sent a handful of astronauts into low earth orbit. In response to this challenge, NASA identified three proposed concepts (shown in Fig. 1) for the mission: earth orbit rendezvous (EOR), lunar orbit rendezvous (EOR), and direct ascent (DA).

In the DA architecture, the entire spacecraft lands on the moon and then launches again for return to the earth. This was initially the leading candidate favored by NASA as it is the simplest, requiring only one dedicated vehicle. However, this single spacecraft necessarily must have a large amount of mass and structure to be able to safely land and re-launch from the moon. Sending this type of payload from orbit would in turn require the development of an enormous launch vehicle.

The EOR architecture employs multiple launches of smaller spacecraft components which are then assembled in earth’s orbit. The assembled vehicle transits to the moon where it lands, and after completion of operations, launches again for return to earth. While ultimately requiring smaller launch vehicles than the DA, the EOR’s major weakness stems from its reliance on both the ability to launch multiple payloads successfully and in turn to assemble a functional vehicle in space. In the early 1960s, there have never been a demonstrated rendezvous of two space vehicles in orbit---let alone the capability to assemble a crewed vehicle.

The final concept, the LOR, requires one launch vehicle that has two components: a lunar lander and a mother ship. Both ships are launched together from a single launch vehicle and enter lunar orbit in tandem. The lander then detaches and descends to the moon. After completion of the mission, the lander ascends for rendezvous with the mother ship. The astronauts transfer from the lander to the mother ship, jettison the lander, and return to earth. This approach, famously championed by John Houbolt, requires a lower overall mass than the EOR and DA since less propellant is required to land and launch the small lunar lander. Similarly, as the lander is abandoned in lunar orbit, the propellant required to return the mother ship to earth is also smaller. As with the EOR concept, however, the disadvantage of LOR is that in the early 1960s, no vehicles had ever performed a rendezvous in space.

The debate over the which architecture to followed led to several intense discussions at NASA. At one point, President Kennedy even had to step in during a press conference to smooth over a heated argument between administrators. Ultimately, however, it was decided that the technical challenges with DA and EOR were too great to be overcome in a decade. This left lunar orbit rendezvous as the down selected architecture, and the Apollo program with the Saturn V launch rocket is the result. The reader can find a nice summary of the story [here](https://www.npr.org/2019/07/18/739934923/meet-john-houbolt-he-figured-out-how-to-go-to-the-moon-but-few-were-listening).

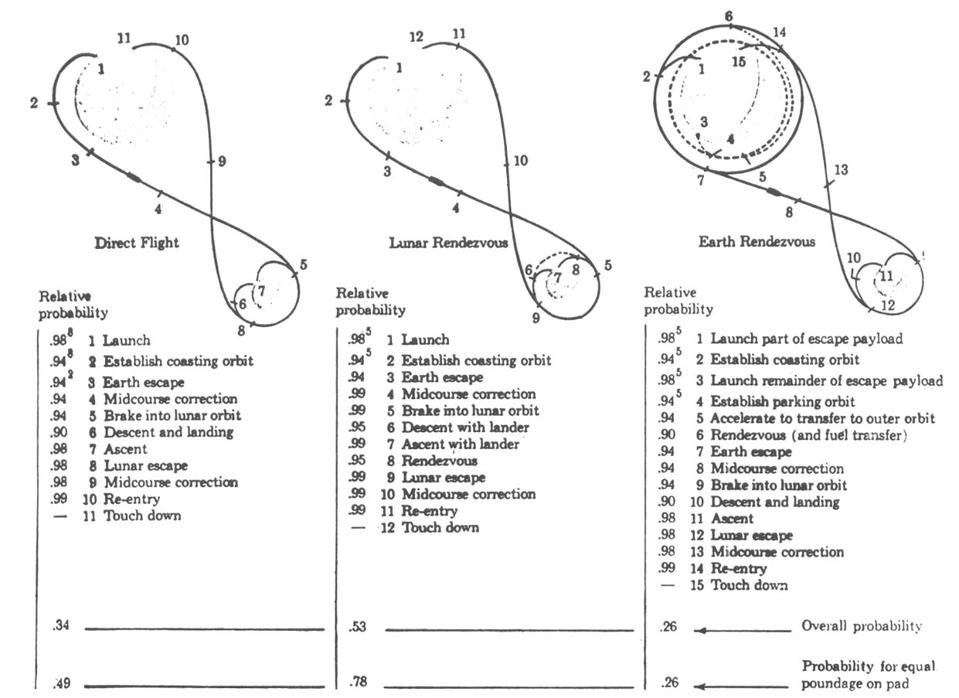


Figure 1: Three proposed architectures for reaching the moon. Image credit: NASA history office.

In this report, we put ourselves in the role of NASA engineers in the early 1960s trying to understand the trade between the direct ascent and a lunar orbit return architecture. We explicitly calculate (subject to some simplifying assumptions) the mass budget for the mission and the required size of the launch vehicles. We then offer a preliminary design for the launch vehicle and simulate its trajectory, illustrating key features in the launch profile.

1. **Key assumptions for analysis**

We outline in the following the key assumptions we use in our trade study comparing the DA and LOR. This includes a discussion of the vehicle dry masses, the delta-v budget, and the propulsion characteristics.

* 1. **Vehicle dry masses**

The LOR has two vehicles for performing in-space maneuvers, a lunar lander and a command and service module (CSM). The DA architecture in contrast has only one vehicle, the CSM. We show in Table 1 the estimated dry masses for these two vehicles. These estimates are based on the required structural mass to support human life for the required mission duration as well as the necessary mass for avionics, propellant storage, and consumables.

Table 1: Dry masses for the CSM and lunar lander

|  |  |
| --- | --- |
| **Vehicle** | **Dry mass** |
| Lunar lander | 4,280 kg |
| Command and service module | 11,900 kg |

* 1. **Delta-v budget**

We show in Table 2 the delta-v requirements for each aspect of the two mission architectures as depicted in Fig. 1. The major difference is that the lunar lander detaches from the CSM for the descent to the moon. This vehicle thus performs the descent and ascent maneuvers. The lander is then jettisoned before the CSM returns to earth. We note that the launch to LEO delta-v requirement is lower than a nominal 7.7 km/s. This stems from the fact that we assume the vehicle will launch due East from Cape Canaveral, thereby starting out with an initial velocity (from the earth’s rotation) with respect to the center of the earth of 0.4 km/s.

Table 2: Delta-v requirements for both mission architectures

|  |  |  |  |
| --- | --- | --- | --- |
| **Maneuver** | **Delta-v (km/s)** | **Vehicle for DA** | **Vehicle for LOR** |
| Launch to LEO and establish coasting orbit | 7.3 | Launch vehicle | Launch vehicle |
| Earth escape | 3.2 | 3rd stage of launch vehicle | 3rd stage of launch vehicle |
| Midcourse corrections | ~0 km/s | N/A | N/A |
| Brake into lunar orbit | 0.9 | CSM | CSM |
| Descent and landing | 2.5 | CSM | Lunar lander |
| Ascent | 2.2 | CSM | Lunar lander |
| Rendezvous | 0.1 | N/A | Lunar lander |
| Lunar escape | 0.9 | CSM | CSM |

* 1. **Average properties or propulsion systems used for maneuvering**

We show in Table 3 the expected specific impulse for the different stages of the mission. For the in-space maneuvers performed by either the CSM or lunar lander, we have assumed a propellant based on N2O4 oxidizer and Aerozine-50 fuel. This choice of propellant balances the need for high fuel economy and the need for a high reliability. As a bipropellant (fuel and oxidizer), this mixture has a higher specific impulse than the common alternative used for in-space propulsion, hydrazine-based monopropellants. The other advantage of this propellant is that it is hypergolic. It will spontaneously ignite when the oxidizer and fuel are combined. This eliminates the need for an ignition system, thereby greatly enhancing reliability (if the ignitor failed when trying to launch off the moon, the astronauts would be stranded!).

Table 3: Assumed average properties of the vehicles

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Propellant** | **Vehicle** | **Average specific impulse** | **Structural coefficient ()** | **Thrust to weight at ignition** |
| N2O4/Aerozine-50 | CSM and lunar lander | 311 s | N/A | N/A |
| LOX/RP-1 | 1st stage | 283 s | 0.05 | 1.2 |
| LOX/LH2 | 2nd stage | 311 s | 0.07 | 0.7 |
| LOX/LH2 | 2nd or 3rd stage | 421 s | 0.19 | 0.5 |

For the launch vehicle stages, we baseline a kerosene first stage (LOX/RP-1) and liquid hydrogen (LOX/LH2) upper stages. Kerosene propellant has a lower upperbound in achievable specific impulse compared to liquid hydrogen. It therefore has a lower fuel economy, which translates to more propellant required for a given maneuver. Indeed, for a delta-v ranging from 0 – 12 km/s, a rocket with RP1 will require on average 1.35 times more mass of propellant than an engine with liquid hydrogen. However, the LOX/RP-1 mixture has a density that is 3 times higher than the LOX/LH2 mixture. As a result, a launch vehicle using LOX/RP-1 will require approximately 4 times less volume for propellant storage than a vehicle with LOX/LH2. This is a critical consideration for the lower stage which spends most of its time climbing out of the atmosphere. The increased volume of the rocket (and by extension) cross-sectional are becomes a prohibitively large source drag for a LOX/LH2 first stage. Once above atmosphere where the smaller, upper stages are ignited, drag profile is no longer a concern. The higher performance LOX/LH2 is therefore the choice for these upper stages.

The estimates for specific impulse in Table 3 for this initial trade study represent average values expected for each stage. For example, state of the art RP-1 based rockets (see Table 4) have specific impulses that range from 263 s at sea level to 304 s at vacuum conditions. As we expect this first stage to climb through the atmosphere to reach the barrier of space, we use a value that represents the average of the two environmental extremes, 283 s. For the second stage with its LOX/LH2 rocket, which may start in atmosphere and then continue to climb into space, we similarly use, 311 s, an average value between the sea level performance (200 s) and vacuum performance (421 s) For the third, LH2 stage, which ignites when the rocket is in space, we assume the vacuum value of specific of 421 s.

We include in Table 3 estimates for the structural coefficients for the launch vehicles. These represent the ratios of structural mass in each stage to the total wet mass of the stage. As noted above, since RP-1 is a denser propellant, it requires less volume for storage. The storage tanks consequently are smaller allowing for lower structural coefficient. Conversely, LH, requires larger volume and more tank mass for its storage. Similarly, since the pressure differential in the tanks will grow higher with altitude, there is a need for more structural mass on the 3rd stage.

Finally, we show in Table 3 the requirements for the thrust to weight of each vehicle stage. For the first stage, the initial trajectory is vertical. The rocket therefore must provide sufficient acceleration (thrust to weight greater than 1) to overcome the full force of gravity. In subsequent stages as the rocket trajectory gains altitude and shifts its trajectory to horizontal with respect to the surface of the earth, the requirements for acceleration relax. The lower thrusts have the additional benefits of lowering the effective g-loading on the astronauts.

* 1. **Baselined engines for trajectory analysis**

We show in Table 4 the properties of the two real engines that were under development at the beginning of the Apollo program, the RP-1 based F-1 engine and the LH2 based J-2. The F-1 interestingly was developed before the 1960s—before there were even designs for a launch vehicle. The J-2 was developed in parallel with the initial formulation of the Apollo program. While we use the average performance metrics in Table 3 to evaluate our initial design, we return to the properties shown in Table 4 to estimate the number of engines and total thrust levels required for each stage.

Table 4: Properties of the engines considered for the rocket stages.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Engine** | **Propellant** | **Sea level thrust (kN)** | **Vacuum thrust (kN)** | **SL Isp (s)** | **Vacuum Isp (s)** | **Mass of engine (kg)** | **Engine diameter** |
| F1 | LOX/RP-1 | 6,770 | 7,770 | 263 | 304 | 8400 | 3.7 m |
| J-2 | LOX/LH2 | 486.2 | 1,033 | 200 | 421 | 1788 | 2.1 m |

# Analysis

We present in the following our design and sizing for both the payload (CSM and Lunar Lander wet masses)

as well as the launch vehicle.

# Payload mass

We calculate here the payload mass of the launch vehicle for both the DA and LOR architectures. This estimate includes the total wet mass, i.e. propellant and dry contributions, of the vehicle(s) that will be sent on a translunar injection to the moon. These estimates are based on using the delta-v budget shown in Table 1, dry masses indicated in Table 2, and the performance metrics for the Aerozine-50 propulsion system shown in Table 3.

* *Include relevant equations and describe what they mean and where they come from*
* *Describe process of calculation*
* *Tabulate final results in Table 5*
* *Discuss why the lunar orbit rendezvous payload mass is smaller*
* *Delete these bullet points when ready to submit report*

# Direct ascent (5 pts)

# Lunar orbit rendezvous (10 pts)

Table 5: Estimated payload masses for DA and LOR architectures

|  |  |
| --- | --- |
| **Architecture** | **Payload mass (kg)** |
| Direct Ascent |  |
| Lunar orbit rendezvous |  |

* 1. **Launch vehicle configuration**

We baseline a three-stage launch vehicle for propelling the payloads shown in Table 5 into cislunar space. This choice represents a compromise between the gains that result from multi-staging and the complexity involved with increasing the number of stages.

* + 1. **Mass of each stage (20 pts)**

The goal of this section is to use the values shown in Table 5 combined with the total delta-v requirement for the launch vehicle (10.5 km/s ) from Table 2 to determine the optimal masses of each stage for both architectures

* *Include relevant equations and describe what they mean and where they come from*
* *Describe process of calculation*
* *Tabulate results in Table 6*
* *Comment on contrast between mass of launch vehicle mass for the DA versus LOR architectures*
* *Delete these bullet points when ready to submit report*

Table 6: Mass of each stage for two architectures

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Architecture** | **1st stage mass (kg)** | **2nd stage mass (kg)** | **3rd stage mass (kg)** | **Total mass including payload (kg)** |
| Direct Ascent |  |  |  |  |
| Lunar Orbital Rendezvous |  |  |  |  |

* + 1. **Rocket sizing (20 pts)**

In this section, we estimate the required thrust levels for the launch vehicle, burn times, mass flow rates, and approximate number of engines to complete each stage. These estimates are informed by the thrust to weight requirements stipulated in Table 3, the actual engine performance shown in Table 4, and the masses reported in Table 6.

For the thrust level at ignition (as reported in Table 3), we assume the first stage F-1 rockets will be at sea level and the upper stage J-2 rockets will both ignite in near vacuum like conditions. We also note that to calculate the actual total thrust of each stage, we first determined the appropriate thrust value based on the required thrust to weight. We then determined the minimum number of engines necessary to provide this thrust (Table 4). We report the value of the thrust given by multiplying this number of engines by the reported thrust per engine. We estimate in this section as well as the diameter of the first stage for both architecture by determining the [optimal circular diameter](https://en.wikipedia.org/wiki/Circle_packing_in_a_circle) that would fit the indicated number of engines.

* *Include relevant equations and describe what they mean and where they come from*
* *Describe process of calculation*
* *Tabulate results in Table 7*
* *Comment on contrast between DA versus LOR architectures*
* *Delete these bullet points when ready to submit report*

Table 7 : Engine characteristics for each architecture

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Architecture** | **Stage** | **Thrust at ignition (kN)** | **Flow rate (kg/s)** | **Burn time (s)** | **Engine type** | **Number of engines** | **Stage diameter (m)** |
| DA | 1st |  |  |  |  |  |  |
| LOR | 1st |  |  |  |  |  |  |
| DA | 2nd |  |  |  |  |  |  |
| LOR | 2nd |  |  |  |  |  |  |
| DA | 3rd |  |  |  |  |  |  |
| LOR | 3rd |  |  |  |  |  |  |

* + 1. **Engine down-select (5 pts)**
* *Justify, using previous results, the choice to baseline the LOR architecture.*
* *Discuss your final LOR design in comparison to the actual specifications for the Saturn V. Are there any differences. Why do you think there are these differences?*
* *Delete these bullets when submitting final report*
  1. **Simulated rocket launch (30 pts)**

We simulate in this section a 2D launch profile for our LOR rocket. For simplicity, we assume

* The initial part of the trajectory is the result of a gravity turn
* After the gravity turn is complete, we gimbal the rocket thrust vector to provide a constant pitch angle, with respect to the local horizon
* We neglect the rotation of the earth.
* We neglect atmospheric drag and lift and solve for the trajectory in 2D
* We assume the mass flow rate through the engines for each stage is constant during the burn (Table 7). However, the thrust level and specific impulse will change because the pressure is varying.
* We target a destination altitude of 200 km by adjusting the gimbal angle of the thruster.

The launch concept of operations is shown in Table 8. We note we solve the trajectory up until (7.3 km/s). This leaves sufficient propellant mass remaining on board to perform the last orbital insertion maneuver (3.2 km/s). The third stage engine cutoffs at this point and will-re-ignite later for the escape trajectory. The model for atmospheric pressure for this calculation is shown in the Appendix.

* *Include relevant equations and describe what they mean and where they come from*
* *Describe process of calculation and how you select the pitch angle*
* *How closely does the mass left over after the burn match what was calculated in parts A and B? Discuss why this is the case.*
* *Plot results in Fig. 2, clearly labeling the events described in Table 8*
* *Delete these bullet points when ready to submit report*

Table 8: Concept of operations for launch profile of rocket

|  |  |
| --- | --- |
| **Mission time** | **Event** |
| 0 s | Vertical liftoff |
| 12 s | Pitching maneuver from vertical. Rocket starts gravity turn |
| 1st stage burn time | Main engine cut out (MECO) and separation |
| 0 s | 2nd stage ignition |
| 0 s | Rocket angled to pitch at with respect to local horizon |
| 2nd stage burn time | Main engine cut out (MECO) and separation |
| 0 s | 3rd stage ignition |
| 3rd stage cutoff | Velocity reaches 7.3 km/s |

A screenshot of a cell phone

Description automatically generatedFigure 2: Characteristics of calculated launch profile. The different events from Table 8 are also labeled.

1. **Discussion and Conclusion (5 pts)**

* *Summarize findings*
* *Discuss launch profile and any challenges (e.g. too much g-loading) and how this might be fixed in real design*
* *Look up and compare results to actual Saturn V launch profile. Discuss differences.*
* *Discuss next steps for refining design*

1. **Appendix**

A screenshot of a cell phone

Description automatically generated Table 9: U.S. standard atmospheric model

Include copies of any code generated here.